LUCUS A NON LUCENDO.

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I reproduce here a curious piece of mathematical legerdemain which proves without proving. It has been, with its variations, the standard for many years in most all the text books. I think it will be difficult to find a proof (?) more utterly lacking in every pedagogical requirement, and yet correct in its final conclusion.

THEOREM. *Any continuous arc of the evolute is equal to the difference between the radii of curvature of the involute that are tangent to the arc at its extremities.*

If r is the radius of curvature and a, b the coördinates of the center of curvature, then the circle of curvature is

$$(x-a)^2+(y-b)^2=r^2 \dots (1).$$

But for a normal through a, b we get

$$y-b=-\frac{dx}{dy}(x-a) \dots (2),$$

and since the normal is tangent to the evolute

$$y-b=\frac{db}{da}(x-a) \dots (3).$$

From (1) and (2), if we suppose the point x, y to move along the curve, and therefore y, a, b, and r to be functions of x, we get

$$(x-a)dx+(y-b)dy-(x-a)da-(y-b)db=rdr,$$

and from (2), $(x-a)dx+(y-b)dy=0$, whence

$$(x-a)da+(y-b)db=rdr \dots (4).$$

From (3) and (4), $(x-a)\dfrac{da^2+db^2}{da^2}=-rdr \dots (5).$

From (1) and (3), $(x-a)^2\dfrac{da^2+db^2}{da^2}=r^2 \dots (6).$

(5) squared, divided by (6), gives $da^2+db^2=dr^2=ds^2$.

Whence $ds=\pm dr \dots (7)$. Q. E. D.

Now, to begin with, calling r the radius of curvature does not make it such. We might just as rightfully call it *not* the radius of curvature. Under this latter hypothesis what does equation (7) prove?

To call such a sequence of *disjecta membra* a proof is little short of absurdity, however correct the formal algebraic operations may be. One seldom runs across such a total lack of syllogistic sequence of thought presented under the guise of proof.

Let us see what the r of equation (7) really does represent.

(1) puts a, b at the center of a circle through x, y.

(2) puts *another* a, b on the normal through the locus of another x, y.

(3) puts a third x, y on the tangent to the locus of a, b.

(4)=1st condition+2d condition. Unites the first and second x, y, and the first and second a, b.

(5)=4th condition+3d condition, puts a, b on the normal to x, y, and at the center of circle r, and *tangent to the normal*, and *therefore* on the evolute, at the center of curvature, and the arbitrary r now becomes the definite R, the radius of curvature, and the a, b becomes α, β, the coördinates of the center of curvature.

(6)=1st condition+3d condition, puts a, b at center of circle through x, y and tangent to the *secant* through x, y, and leaves r and a, b arbitrary.

(7)=5th condition+6th condition, converts the r of (6) into the R of (5), and the a, b into α, β, and hence (7) becomes $\sqrt{(d\alpha^2+d\beta^2)}=dS=\pm dR$, and the theorem follows.

It will be noticed that there is not a suspicion of anything pertaining to the evolute until we reach equation (5), and that then by conjunction of conditions the thing snaps into place and the a, b locus becomes, *willy nilly*, the α, β locus or evolute, and the r similarly becomes R, the radius of curvature. The terminology applied to a, b, r is ineffective. It is the conjunction of conditions that decide what they should be called.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

142. **Proposed by M. J. CRAWFORD, Principal of Crawford's Academy, Savannah, Ga.**

A gentleman has a garden 400 feet long and 300 feet wide, which he wishes to raise 9 inches higher by means of the earth to be dug out of a ditch 6 feet wide and surrounding the entire garden. How deep must the ditch be?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$400\times300\times\frac{9}{12}=90000$ cubic feet of earth necessary to raise the garden.

$2(412+300)\times6=8544$ square feet, area of surface of ditch.

$90000\div8544=10\frac{95}{178}$ feet, depth of ditch.